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**G. Murugusundaramoorthy and P. Usha**

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**B. K. De**

ON QUASI-EINSTEIN SPACES

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**Sevtap Sümer Eker and Shigeyoshi Owa**

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with negative coefficients. Relevant connections with various known integral means inequalities are also pointed out.

**B. Srutha Keerthi, B. Adolf Stephen, A. Ganghadaran  
and S. Sivasubramanian**

SOME PROPERTIES OF HYPERGEOMETRIC FUNCTIONS FOR  
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**A. A. Shaikh, Sanjib Kumar Jana and S. Eyasmin**

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**P. Chandrakala and S. Antony Raj**

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method. Transient and steady-state velocity and temperature profiles, the local as well as average skin friction and the Nusselt number are shown graphically. The effects of heat transfer for different parameters like magnetic field parameter, radiation parameter, Prandtl number and thermal Grashof number are studied. It is observed that the number of steps for convergence to steady state depends strongly on  $Gr$ .

**George A. Anastassiou**

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**Abstract:** Here we present Poincaré type fractional inequalities involving fractional derivatives of Canavati, Riemann-Liouville and Caputo types. The results are general  $L_p$  inequalities forward and converse, univariate and multivariate on a spherical shell. We give applications to ODE and PDE. We present also mean Poincaré type fractional inequalities.

**Sergiu I. Vacaru and Juan F. González-Hernández**

NONLINEAR CONNECTIONS ON GERBES, CLIFFORD-FINSLER  
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**Abstract:** The geometry of nonholonomic bundle gerbes, provided with nonlinear connection structure, and nonholonomic gerbe modules is elaborated as the theory of Clifford modules on nonholonomic manifolds which positively fail to be spin. We explore an approach to such nonholonomic Dirac operators and derive the related Atiyah–Singer index formulae. There are considered certain applications in modern gravity and geometric mechanics of Clifford–Lagrange/ Finsler gerbes and their realizations as nonholonomic Clifford and Riemann–Cartan modules.

**C. S. Bagewadi, D. G. Prakasha and Venkatesha**

A STUDY OF RICCI QUARTER-SYMMETRIC METRIC  
CONNECTION ON A REIMANNIAN MANIFOLD

607-615

**Abstract:** The paper is dealt with the study of Einstein manifold admitting Ricci quarter-symmetric metric connection  $\bar{\nabla}$  and several interesting results are obtained.

**Guangfeng Liu and Xianyi Li**

GLOBAL ATTRACTIVITY OF A NONLINEAR DIFFERENCE  
EQUATION

617-627

**Abstract:** In this paper, we study the global asymptotic behavior of the following rational difference equation

$$y_{n+1} = \frac{r + py_n + y_{n-1}}{qy_n + y_{n-1}}, \quad n = 0, 1, 2, \dots,$$

where the parameters  $p, q, r \in (0, \infty)$ , and the initial conditions  $y_{-1}, y_0$  are positive real numbers. Our results partly show that local stability of the positive equilibrium of the equation implies global stability, which partly affirms Conjecture 10.5.2 in [M. R. S. Kulenovic and G. Ladas, Dynamics of Second Order Rational Difference Equations, 2002].

**Donal O'Regan**

FIXED POINT THEORY IN FRÉCHET SPACES FOR PERMISSIBLE  
URYSOHN TYPE MAPS

629-646

**Abstract:** New fixed point theorems for multivalued maps between Fréchet spaces are presented. We discuss first  $J$  maps, then admissible maps and finally permissible maps. In particular we use degree and index theory and a Lefschetz fixed point theorem to obtain applicable fixed point theorems in Fréchet spaces for general classes of maps.

**Fu-Gui Shi**

A NEW APPROACH TO FUZZY S-CLOSEDNESS

647-661

**Abstract:** A new definition of S-closedness is presented in  $L$ -topological spaces when  $L$  is a complete DeMorgan algebra. It is defined by means of semiopen  $L$ -sets and their inequality. This definition does not rely on the structure of the basis lattice  $L$  and no distributivity in  $L$  is required. It can also be characterized by semiclosed  $L$ -sets, regularly closed  $L$ -sets, regularly open  $L$ -sets, regularly semiopen  $L$ -sets, regularly semiclosed  $L$ -sets and their inequalities. When  $L$  is a completely distributive DeMorgan algebra, its many characterizations are presented.

**G. R. Hiremath**

A NOTE ON DEVELOPABILITY OF  $w\Delta$  SPACES AND  
METRIZABILITY OF  $wM$  SPACES

663-670

**Abstract:** A property designated as semi- $(\mathbb{C}_2)$  that a large class of topological spaces including the Moore spaces satisfy, is discovered and in terms of this property the new characterizations of Moore spaces and metrizable spaces are established:

1. A topological space is a Moore space if and only if it is a  $w\Delta$  space with the semi- $(\mathbb{C}_2)$  and the quasi  $(\alpha_1)$  properties.
2. A topological space is a Moore space if and only if it is a  $T_1$  quasi-developable  $\beta$  space with the semi- $(\mathbb{C}_2)$  property.
3. A topological space is a metrizable space if and only if it is a  $wM$  (a fortiori,  $M$  or countably compact) space with the semi- $(\mathbb{C}_2)$  and the quasi  $(\alpha_1)$  properties.

**Edward Beckenstein and Lawrence Narici**

WEIGHTED OPERATORS IN ORLICZ AND LORENTZ SPACES

671-690

**Abstract:** Let  $X$  and  $Y$  be real or complex Banach spaces with Schauder bases  $\{x_n\}$  and  $\{y_n\}$ , respectively. If  $x = \sum_{n \in \mathbf{N}} (x(n)) x_n$

and  $y = \sum_{n \in \mathbf{N}} (y(n)) y_n$  are such that  $x(n) y(n) = 0$  for every  $n$ ; we write  $x * y = 0$ . An additive map  $H : X \rightarrow Y$  such that  $x * y = 0 \Rightarrow Hx * Hy = 0$  is called a *basis separating map*. By considering this limited form of multiplication-preservation, we utilized some function space-Banach algebra techniques to obtain automatic continuity and open mapping type results in [7] and [8]. Specifically, we developed a canonical form for basis separating maps which under certain circumstances reduces to what we call a *weighted composition* [Definition 3.3]. In order to define a weighted permutation, we need to know, given  $h : \mathbf{N} \rightarrow \mathbf{N}$ , for what nonvanishing sequences (**weights**)  $w : \mathbf{N} \rightarrow \mathbf{K} = \mathbf{R}$  or  $\mathbf{C}$  does  $\sum_{n \in \mathbf{N}} w(n) x(h(n)) y_n$  converges for all  $x = \sum_{n \in \mathbf{N}} x(n) x_n \in X$ ? We obtain some general conditions under which  $w$  is a weight for maps between  $\ell_p$  spaces (Theorem 4.1), Lorentz spaces (Theorem 5.6), and Orlicz spaces (Theorems 6.7, 6.11 and Corollary 6.12).