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**H. W. Gould and Temba Shonhiwa**

FUNCTIONS OF GCD'S AND LCM'S

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**Abstract:** In this paper we investigate the function  $\gamma(n) = \sum_{k=1}^n [k, n]$  and its generalizations. We also obtain results on the generalized Euler phi function

$$\phi_f(n, m, a) = \sum_{\substack{1 \leq k \leq n \\ (k, a) = m}} f(k).$$

**Pentti Haukkanen and Jun Wang\***

HIGH DEGREE ANALOGS OF MENON'S IDENTITY

37-42

**Abstract:** P. Kesava Menon's elegant identity states that

$$\sum_{\substack{a \pmod{n} \\ (a, n) = 1}} (a - 1, n) = \phi(n)\tau(n)$$

where  $\phi(n)$  is Euler's totient function and  $\tau(n)$  is the number divisors of  $n$ . This identity has been generalized in various directions in the literature. In this note we give analogs of this identity arising from the concept of a  $k$ -dimensional  $r$ -th degree reduced residue system modulo  $n$ .

**P. Jeganathan**

ON SOME CHARACTERIZATIONS OF TAUBERIAN RELATIONS

43-57

**Abstract:** A known theorem states that if  $B \subset X$  is a bounded set and if  $T$  is a linear relation between  $X$  and  $Y$ , where  $X$  and  $Y$  are normed spaces, then  $T$  is Tauberian iff  $TB$  relatively weakly compact implies that  $B$  is relatively weakly compact. In this paper various applications of this results are shown under the setting of linear relations. In addition some characterizations of Tauberian relations are derived.

**KJ. Maniharsingh**

THEOREM RESEMBLING BIRKHOFF'S FOR ELECTROMAGNETIC FIELDS  
IN SELF-CREATION SCALAR TENSOR THEORY

59-64

**Abstract:** A result analogous to that of Birkhoff's theorem of general relativity has been established to exist for electromagnetic fields in a self-creation theory of gravitation proposed by Barber, when the scalar field introduced in the theory is independent of time. Some important investigations are made in this connection and it seems that continuous creation of matter in such a case is considerably slow. New vistas towards the evolution of our Universe is expected from such a study.

**Sanjai Rai and C. C. A. Sastri**

ON THE SIMPLIFICATION OF OVSIANNIKOV'S METHOD FOR THE  
CONSTRUCTION OF PARTIALLY INVARIANT SOLUTIONS

65-74

**Abstract:** The simplification of Ovsiannikov's method for the construction of partially invariant solutions (PIS) of a system of partial differential equations (pde<sup>s</sup>) carried out earlier by Sastri, Dunn and Rao (1987) in the case of one dimensional heat equation  $u_1 = u_{xx}$  is extended here to systems of nonlinear pde<sup>s</sup>. Although a procedure that works for all systems of pde<sup>s</sup> is not given here, the examples considered, namely the equations of the transonic flow of a gas, the Landau damping equation and the Burgers equation provide a basis for handling nonlinear equations in general. Invariant and partially invariant solutions are constructed for two different forms of the Burgers equation. Both type of solutions exhibit the blow up property. The PIS, however, turns out to be reducible, i.e., invariant w.r.t. a subgroup of the symmetry group. The problem of finding irreducible PIS for some form of the generalized Burgers equation will be addressed in a future publication.

**G. Criscuolo and L. Scuderi**

CONVERGENCE OF PRODUCT QUADRATURE RULES WITH  
PREASSIGNED NODES

75-90

**Abstract:** Product quadrature rule, with preassigned and simple nodes, to compute ordinary integrals, is investigated. Nodes are located within the interval of integration. The convergence of the proposed formula is proved and an estimate in  $L_1$ -weighted norm is established.

**Ming Po Chen and Sehie Park**

ON THE BROWDER TYPE BEST APPROXIMATION THEOREMS

91-97

**Abstract:** Best approximation theorems extending Browder's sharpened form of the Schauder fixed point theorem are obtained. Our new results extend and unify many known theorems.

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