

# Bulletin of the Allahabad Mathematical Society

Volume 34, Part 2, 2019

---

## CONTENTS

**Saurabh Porwal, Poonam Sharma and Omendra Mishra**

HARMONIC MULTIVALENT FUNCTIONS WITH POSITIVE COEFFICIENTS 147-158

**Abstract:** In the present paper, we introduce and study a new subclass  $RS_H^p(n, \lambda, \beta)$  of harmonic multivalent functions by using multiplier transformation. We determine coefficient estimates, extreme points, distortion bounds, convolution properties and convex combination for the above class of harmonic functions and discuss a class preserving integral operator.

**Shantanu Bhaumik and Surendra Kumar Tiwari**

FIXED POINT THEOREMS FOR MAPPINGS SATISFYING INTEGRAL TYPE  
 $F$ -CONTRACTION IN METRIC SPACES ENDOWED WITH A GRAPH 159-172

**Abstract:** The purpose of this paper is to find the condition for which a function satisfying integral type  $F$ - $G$ -contraction defined on a metric space will be Picard operator. With suitable examples we prove that an integral type  $F$ - $G$ -contraction is more general than that of an  $F$ -contraction and  $G$ -contraction.

**Mukesh Kumar Singh and Saurabh Porwal**

SOME SUFFICIENT CONDITIONS FOR AN INTEGRAL OPERATOR INVOLVING  
BESSEL FUNCTIONS 173-180

**Abstract:** The purpose of the present paper is to obtain some sufficient conditions for starlikeness of an integral operator involving Bessel functions of the first kind. Relevant connections of the results presented here with various known results are briefly indicated.

**T. D. Narang and Sahil Gupta**

ON CHEBYSHEV CENTERS 181-199

**Abstract:** In this paper, we survey the main known results on Chebyshev centers and point out some of the connections between the theory of Chebyshev centers and other problems of Approximation Theory and of Banach space theory. We also discuss results on the existence, uniqueness and nature of Chebyshev center points, continuity of the Chebyshev center map and conditions under which bounded sets are centrabable.

**Om P. Ahuja, S. Sivasubramanian, Y. Polatoğlu and A. Çetinkaya**  
CERTAIN SUBCLASSES OF BI-UNIVALENT FUNCTIONS GENERATED BY  
CHEBYSHEV POLYNOMIALS AND  $q$ -DERIVATIVE OPERATOR 201-216

**Abstract:** In this paper, we introduce and investigate new subclasses of the function class  $\Sigma$  of bi-univalent functions defined in the open unit disc, which are associated with  $q$ -difference operator by means of Chebyshev polynomials. In particular, we find the initial estimates of  $|a_2|$  and  $|a_3|$  for functions in the new subclasses introduced here. We also obtain Fekete-Szegő inequalities for these classes. Several (known or new) consequences of the results are also obtained.

**Sandra Pinelas and Paolo Emilio Ricci**  
CIRCULAR BASED SHEFFER POLYNOMIAL SEQUENCES 217-234

**Abstract:** In this article we introduce Sheffer polynomial sequences whose characteristic elements are based on circular or inverse circular function and we derive their main properties, including recurrence relations and differential equations. The article is part of an attempt to sort Sheffer's polynomial sequences on the basis of the characteristics of their generating function. In previous articles, Sheffer sequences arising from other types of generating functions, based on powers or exponentials have been introduced.

**Feng-Zhen Zhao**  
LOG-BEHAVIOR FOR SOME SEQUENCES RELATED TO CAUCHY NUMBERS  
OF TWO KINDS 235-242

**Abstract:** In this paper, we discuss log-behavior for some sequences related to Cauchy numbers of two kinds. For Cauchy numbers of two kinds  $\{a_n\}_{n \geq 0}$  and  $\{b_n\}_{n \geq 0}$ , we prove that two sequences  $\{|a_n|\}_{n \geq 1}$  and  $\{b_n\}_{n \geq 0}$  are 2-log-convex. In addition, we show that sequences  $\{|a_{n+1}| - |a_n|\}_{n \geq 2}$ ,  $\{b_{n+1} - b_n\}_{n \geq 1}$  are log-convex, and  $\{|a_n|\}_{n \geq 1}$  and  $\{b_n\}_{n \geq 0}$  are ratio log-concave.

**Eskandar Ameer and Muhammad Arshad**  
FIXED POINT RESULTS OF GENERALIZED  $(\psi, \phi)$ -RATIONAL TYPE  
CONTRACTIONS WITH APPLICATION TO NONLINEAR MATRIX EQUATIONS 243-261

**Abstract:** The purpose of this paper is to introduce the notion of generalized  $(\psi, \phi)$ -rational type contractions and establish some new fixed point theorems for such mappings in complete metric spaces. Our results are an extension and improvement of Jleli and Samet fixed point theorem, Wardowski fixed point theorem, Piri and Kumam fixed point theorem, Geraghty fixed point theorem and Liu et al. fixed point theorem. We give an example for supporting these results. Finally, an application is given to set up the existence of positive definite solution of non-linear matrix equation.

**James F. Peters**

VORTEX NERVES AND THEIR PROXIMITIES. NERVE BETTI NUMBERS  
AND DESCRIPTIVE PROXIMITY

263-276

**Abstract:** This article introduces vortex nerve complexes in CW (Closure finite Weak) topological spaces, which first appeared in works by P. Alexandroff, H. Hopf and J.H.C. Whitehead during the 1930s. A *vortex nerve* is a CW complex containing one or more intersecting path-connected cycles. Each vortex nerve has its own distinctive shape. Both vortex nerve shapes (bounded planar surfaces with nonempty interior) and holes (bounded planar surfaces with empty interior that live inside and define shapes) have boundaries that are path-connected cycles. In the context of CW complexes, the usual Betti numbers  $\mathcal{B}_0$  (cell count),  $\mathcal{B}_1$  (cycle count) and  $\mathcal{B}_2$  (hole count) provide a basis for the introduction of several new Betti numbers, namely, vortex  $\mathcal{B}_{vtx}$ , vortex nerve  $\mathcal{B}_{vNrv}$  and shape  $\mathcal{B}_{sh}$  introduced in this paper. In addition, results are given for CW complexes equipped with a descriptive proximity as well as for the homotopy types of vortex nerves and the complexes and cycles contained in the nerves.

**Yogesh J. Bagul and Christophe Chesneau**

NEW SHARP BOUNDS FOR TANGENT FUNCTION

277-282

**Abstract:** The aim of this paper is to prove new and sharp bounds for the function  $\frac{\tan x}{x}$ , improving some existing bounds in the literature.

\*\*\*\*\*