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CONTENTS

Abstract: The aim of this paper is to establish a unique common fixed point theorem for two pairs of occasionally weakly compatible single and multi-valued maps in a metric space. This result generalizes, improves and extends the result of Türkoğlu et al. [6] and references therein.

Abstract: Let C be a nonempty closed convex subset of a real Hilbert space H and T be a strong asymptotically regular asymptotically nonexpansive mappings from C into itself with nonempty fixed points set $F(T)$. Let $\{x_n\}$ be modified Mann iterative process in C defined by $x_{n+1} = \alpha_n x_n + (1-\alpha_n)T^n x_n$ for any given initial guess $x_1 \in C$ and all $n \geq 1$, where $\{\alpha_n\}$ is a sequence in $[0,1)$ with condition $\sum_{n=1}^{\infty} \alpha_n(1-\alpha_n) = \infty$. Then $\{x_n\}$ converges weakly to a fixed point $p \in F(T)$.

Abstract: We define and characterize $\mathcal{I}$-locally $\star$-closed and $\mathcal{I}_g$-locally $\star$-closed sets and discuss their properties.

Abstract: Every Jordan generalized k-derivation of a Γ-ring is not a generalized k-derivation of the same. In this paper, we show that under some conditions every Jordan generalized k-derivation of a 2-torsion free completely semiprime Γ_N-ring is a generalized k-derivation by developing a number of results relating to these derivations of certain Γ-rings.

Abstract: We introduced a new subclass of analytic functions making use of Dziok-Srivastava operator in the open unit disc. A sufficient condition and some inclusion relations associated with (n, δ) neighborhoods of the functions belonging to the class are obtained.

Abstract: The main object of the present paper is to give an application of Nunokawa's Lemma.

Abstract: Let $G = (V, E)$ be a connected graph. A subset X of E is called an edge dominating set of G if every edge in $E - X$ is adjacent to some edge in X. An edge dominating set X is called a connected edge dominating set if the edge induced subgraph $\langle X \rangle$ is connected. The minimum cardinality of a connected edge dominating set of G is called the connected edge domination number of G and is denoted by γ'_c. The maximum order of a partition of E into connected edge dominating sets is called the connected edge domatic number of G and is denoted by d'_c. In this paper we obtain several results on γ'_c and d'_c.

Abstract: A graph G with q edges is said to be elegant if the vertices of G can be labeled with distinct integers $\{0, 1, 2, \ldots, q\}$ in such a way that the set of values on the edges obtained by the sums (mod $q + 1$) of the labels of their end vertices is $\{1, 2, \ldots, q\}$. A graph with p vertices and q edges is called total edge-magic if there is a bijection $f : V \cup E \to \{1, 2, \ldots, p + q\}$ such that there exists a constant s for any (u, v) in E satisfying $f(u) + f(u, v) + f(v) = s$.

A merge graph $G_1 * G_2$ can be formed from two graphs G_1 and G_2 by merging a node of G_1 with a node of G_2. In this paper, we prove that the merge graphs $P_5 * S_n$ and $P_6 * S_n$ are elegant and some s-${}^nC_m$ graphs and merge graphs are edge-magic.

H. W. Gould

PROOF AND GENERALIZATION OF AN IDENTITY OF E. T. BELL 71-82

Abstract: E. T. Bell posed the following identity as problem 3457 in 1930 in the American Mathematical Monthly:

$$\sum_{k=0}^{n}(-1)^k \binom{n}{k} \binom{n+k}{k} \binom{2k}{k} 2^{2n-2k} = \frac{1+(-1)^n}{2} \binom{n}{n/2}^2 . \quad (1)$$

It appears that no proof or comment was ever published in the Monthly. We prove Bell's identity and also establish the elegant generalization that

$$\sum_{k=0}^{n}(-1)^k \binom{n}{k} \binom{n+k}{k} \binom{2k}{k+j} 2^{2n-2k}$$
$$= \binom{n+j}{\frac{n+j}{2}} \binom{n-j}{\frac{n-j}{2}} \frac{(-1)^n+(-1)^j}{2}, \quad (2)$$

where $-n \le j \le n, n \ge 0$. The formulae were listed in [1] as relations (6.35) and (6.34), respectively. The paper closes with a quick proof of (1) using a hypergeometric series formula of G. N. Watson.

Finally, Bell's identity is inverted to yield the new identity

$$\sum_{0\le k\le n/2} \frac{4k+1}{n+2k+1} \binom{2n}{n-2k} \binom{2k}{k}^2 2^{2n-4k} = \binom{2n}{n}^2 . \quad (3)$$

A similar inversion is given for the generalized Bell identity.

Abstract: This paper aims at presenting in new unified study on multidimensional Voigt functions through generalized confluent hypergeometric functions. Further representations and series expansions involving mutidimensional classical polynomials laguerre and hermite for mathematical physics are established. Generating function of laurant's type are also obtained.

Abstract: The notion of the entropy is of fundamental importance in the different areas such as probability and statistics, communication theory, economics and physics. In this paper, we propose two new generalized classes of measures of uncertainty for a random variable X based on the survival exponential entropies and derive explicit expression of the proposed measures for two parametric exponential and three parametric Weibul distribution. Also its particular cases have been studied.

Abstract: After M. Fontana and J. Huckaba, we study localizing systems and semistar operations on a semigroup.

David E. Dobbs and Jay Shapiro

Abstract: Our main result states that if R is a unital subring of a (commutative unital) ring T such that the canonical map $f^\star$: $\mathrm{Spec}(T) \to \mathrm{Spec}(R)$ is open (relative to the Zariski topology) but not surjective, then some non-nilpotent element of R belongs to each prime ideal of R which is not in the image of $f^\star$. Consequently, an example is given of a principal ideal domain R and an infinite cyclic group G acting (via ring automorphisms) on R such that the fixed ring R^G is a Noetherian unique factorization domain, the ring extension $R^G \subseteq R$ has the going-down property, and $\mathrm{Spec}(R) \to \mathrm{Spec}(R^G)$ is not open. In another example, an action of an infinite cyclic group G on a unique factorization domain R is constructed so that $\mathrm{Spec}(R) \to \mathrm{Spec}(R^G)$ is open but the action is not locally finite.

W. T. Sulaiman

Abstract: Extension of an integral inequalities are presented which improve some earlier as well as recent inequalities.

H. Movahedi-Lankarani and R. Wells

Abstract: The Tychonoff problem is answered in $\mathfrak{L}_{K,1}$ (the category of metric spaces with diameter $\leq K$ and maps with Lip ≤ 1) as follows: For a directed set I and an inverse system

$\mathcal{X} : I \longrightarrow \mathfrak{L}_{K,1}$ with compact factors, the limit space $|\lim(\mathcal{X})|$ is compact if and only if the Hausdorff-Gromov limit of the net $(\mathcal{X}(i))_{i \in I}$ exists and is equal to $|\lim(\mathcal{X})|$. For more general metric categories $\mathfrak{C} \supset \mathfrak{L}_{K,1}$ we introduce co-uniform limits as well as $Y : \mathfrak{C}^I \longrightarrow \mathfrak{L}^I_{K,1}$ in terms of which we obtain: If $\mathcal{X}$ has compact factors, then $|\lim(\mathcal{X})|$ is compact if and only if the Hausdorff-Gromov limit of the net $(Y(\mathcal{X}(i)))_{i \in I}$ exists and is equal to $|\lim(\mathcal{X})|$.

There is also a corresponding Metric Tychonoff Product Theorem: Let S be a set, and for each $s \in S$ let X_s be a nonempty compact metric space. Let $I(S)$ denote the directed set of finite subsets of S ordered by inclusion. Then the metric product $\prod_{s \in S} X_s$ is compact if and only if $\lim_{F \in I(S)} \sup_{s \notin F} \operatorname{diam}\,(X_s) = 0$.

Metric inverse limits usually change when I is replaced with a cofinal subset. For $\mathcal{X} : I \longrightarrow \mathfrak{C}$, we define $\limsup(\mathcal{X})$ as the direct limit of spaces $\left|\lim(\mathcal{X}|_{I[i \leq)})\right|$, where $I[i \leq) = \{j \in I \mid i \leq j\}$. This space is the natural generalization of the metric inverse limit of M. Moszyńska. When the factors are compact, $\limsup(\mathcal{X})$ is compact provided that the bonding maps are continuous and quasi-isometric.

For illustration, the family (parametrized by ε) of metric inverse systems

$$\mathcal{S}(\varepsilon) = (S^1 \xleftarrow{\text{sq}} \varepsilon S^1 \xleftarrow{\text{sq}} \varepsilon^2 S^1 \xleftarrow{\text{sq}} \cdots)$$

where sq is the squaring map, is studied in some detail. For $0 < \varepsilon < 1$, the space $|\lim \mathcal{S}(\varepsilon)|$ is topologically a solenoid and the net $(Y(\mathcal{S}(\varepsilon)(i)))_{i \in I}$ is essentially the net of core circles when the solenoid is expressed as an intersection. Moreover, for $0 < \varepsilon \neq \varepsilon' < 1$ the two limits $|\lim \mathcal{S}(\varepsilon)|$ and $|\lim \mathcal{S}(\varepsilon')|$ are not bi-Lipschitz equivalent.

H. Movahedi -Lankarani and R. Wells

HAUSDORFF-GROMOV INVERSE LIMITS 199-231

Abstract: We introduce the notion of Hausdorff-Gromov inverse limit and compare it to the notions of metric inverse limit and metric inverse lim sup introduced in [15]. For $\mathcal{X}$ an inverse system of compact metric spaces and continuous bonding maps, it is shown that there is a bijective correspondence (given explicitly) between the equivalence classes of compact Hausdorff-Gromov inverse limits for subsystems of $\mathcal{X}$ and the so called uniformly weakly rigid subsets of $\lim(\mathcal{X})$ which are both closed and compact with respect to the pseudometric d_{ls}. We also introduce a natural transformation Q of metric inverse systems to achieve much the same result as with the canonical remetrization transformation $Y(\mathcal{X})$ in [15]: For many inverse systems $\mathcal{X}$ we have

$$\lim_{\text{HGI}} Q(\mathcal{X}) = \lim Q(\mathcal{X}) = \lim(\mathcal{X}) = \lim Y(\mathcal{X}) = \lim_{\text{HGI}} Y(\mathcal{X}).$$

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